(This informal note has nothing to do with a survey which I will write in the near future.)

# FAQ on "Inter-Universality [1]".

Q1. Does the proof contain non-trivial operations related to changing the universe in the sense of the foundations of mathematics or logic?

A1. I do *not* think so (at least at the time of writing). "Changing the universe" is one of the important features in S. Mochizuki's inter-universal Teichmüller theory. However, the proof does *not* rely on non-trivial operations in the sense of the foundations of mathematics or logic. **The essential mathematical tools in the proof are anabelian reconstruction algorithms.**

Q2. Then what do you mean by changing the universe?

A2. In inter-universal Teichmüller theory, the ring structure itself is deformed. This means that we often encounter situations in which we cannot use conventional scheme theory, because scheme theory is based on commutative ring theory. We cannot transport operations or base points from one scheme theory to another. The "labels" assigned to objects in one scheme theory in an arbitrary manner are no longer available in another. We consider this phenomenon as a "change of universe". You may think of it as a change of the Grothendieck universe under consideration in the rigorous sense. Or, you may think of it as a change of the scheme theory under consideration, i.e., working to the end in one Grothendieck universe from the rigorous point of view of the foundations of mathematics. In this new kind of geometry, our main interest lies in the indeterminacies which arise from traveling from one universe/scheme theory to another. Especially, the important things are to control them, to reduce them by some rigidities, to kill them by some operations like taking $p$-adic logarithms for the roots of unity, to estimate them by considering that some images are contained in some containers even though they are not precisely determinable, and to synchronise some indeterminacies to others and so on. In the first place, we need to recognise the existence of the indeterminacies to perform the above things. The notion of "changing the universe" seems useful to make the existence of the indeterminacies clear. I wrote the words "this new kind of geometry" in the above. However, "this new kind of geometry" might be even out of the framework of "geometry" if we stand in the position that a geometry so far treats only the (continuous, differentiable, polynomial etc.) morphisms which are compatible with the ring structures (more precisely, the morphisms of the ringed topoi).

Q3. No need to change the universe if we take at the beginning a large enough universe containing the possible universes we will use?

A3. Certainly, we have no need of changing the universe in this sense. However, even though any compact manifold can be embedded into a Euclidean space of a large enough dimension, it is unnatural to define the notion of compact manifold to be a subspace of a Euclidean space. In the same way, it is much more natural to consider that we have scheme theories locally and we travel from one universe to another than to take at the beginning a large enough universe. Furthermore, it seems psychologically difficult to make the indeterminacies mentioned in A2 clear, and it is very confusing if we work in only one large such universe.

Q4. I don't understand well. Can you give me a simple example like a toy model?

A4. As a toy model, we can consider the Fourier transform as "changing the universe". We consider a (suitable) function $f(x)$ on $\mathbb{R}$ and its Fourier transform $\hat{f}(\xi)$. Then, we can consider

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that both of the domains of the variables $x$ and $\xi$ are the same $\mathbb{R}$. However, strictly speaking, these domains are different in the origins. Another toy model is changing of the coordinates. For example, changing the cusp at $\infty$ and the cusp at 0 by $z \mapsto -\frac{1}{z}$ in the complex upper half plane is the change of the base points of the coordinates, and can be considered as a toy model of "changing the universe". The above example of Fourier transform is nothing but the above example of the change of the coordinates from the point of view of the functional equation of the theta function (more generally, modular forms) $\theta(t) = \frac{1}{\sqrt{t}}\theta\left(\frac{1}{t}\right)$. I wrote a sentence "it is much more natural to consider that we have scheme theories locally and we travel from one universe to another" in A3. The above example of the change of coordinates as a toy model is compatible as well with the point of view that we define a compact manifold as an object made from local objects by patching data of the changes of the coordinates, not as a subspace of a Euclidean space.

The sensitive feeling to distinguish two identifiable objects of different origins is important in inter-universal Teichmüller theory. If we consider two different holomorphic structures on $\mathbb{R}^2$, then both of them are $\mathbb{C}$. We cannot see the difference between them if we consider only the holomorphic structures. However, if we consider the underlying $\mathbb{R}^2$, then we can see the non-holomorphic relation between them and can measure the difference between them. We study a similar thing in inter-universal Teichmüller theory, that is, we deform the arithmetic holomorphic structure (= ring structure) of a number field non-scheme-theoretically, and we measure the difference, which cannot be seen from scheme theoretic point of view, between the two arithmetic holomorphic structures, which are isomorphic as abstract rings, by introducing "mono-analytic" point of view. In inter-universal Teichmüller theory, we call a property to be **uniradial** if it makes sense only in one arithmetic holomorphic structure, to be **coric** if it is a common property among all other arithmetic holomorphic structures, to be **multiradial** if it can be described in terms of all other arithmetic holomorphic structures.

Q5. Does one *really* need to change the universe for a proof of the abc conjecture?

A5. If we use the meanings of "changing the universe" and "need" in the sense explained above, then I do not know, at least at the time of writing, of any proof that does not require one to change the universe.

N.B.1: This informal note is just my opinion. There is a possibility that it does not precisely reflect Shinichi Mochizuki's thought.
N.B.2: This informal note concerns only the matters around the "inter-universality", not on the details of the theory. Especially, I wrote it mainly for the purpose of disabusing of the misunderstanding like Q1. For the details of the theory, please wait for the survey I will write in the near future.
N.B.3: At the time of writing (April/2013), the papers of inter-universal Teichmüller theory are under the checks of the details. **I have *no* intention by this informal note to claim or to guarantee that the papers are correct at the time of writing (April/2013).** I refuse all of the interviews from the mass media until the situation around the papers will be stabilised.

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